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Invariant formulation of CP violation for four quark families

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Abstract

We find a minimal set of constraints which are independent of the choice of weak quark basis and necessary and sufficient for CP conservation for four quark families, including also the case of degenerate quark masses. These invariant conditions are written in the mass eigenstate basis as a function of the fermion masses and charged current mixings. CP violation is then related to the areas of three unitarity quadrangles and the CP violating effects of the fourth family are discussed in the case of small mixings.

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1 Introduction.

The observed CP violation in the K^0-$\bar{K}^0$ system is accounted for in the standard model by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. There is only one CP violating phase for three families of quarks. If a fourth family exists, there are three CP violating phases [2]. Although the Z^0 invisible width excludes a fourth generation with a light neutrino [3], an extra heavy family is not ruled out [4]. In fact, the deviation from the pattern of CP violation expected for three families, for instance, in b physics may signal to a fourth quark generation [5].

The CKM matrix is defined in the quark mass eigenstate basis and therefore, it is not invariant under arbitrary unitary transformations of the quark basis. For three fermion families it has been identified a quantity invariant under quark basis transformations whose vanishing characterizes CP conservation [6]:

$$I \equiv \det\, [M_u M_u^\dagger, M_d M_d^\dagger] = 0, \tag{1}$$

where $M_{u,d}$ are the up and down mass matrices. This invariant formulation makes apparent the necessary and sufficient conditions for CP conservation (i). Thus

$$\begin{aligned} I &= -2i(m_t^2-m_c^2)(m_t^2-m_u^2)(m_c^2-m_u^2)(m_b^2-m_s^2)(m_b^2-m_d^2)(m_s^2-m_d^2) \\ &\times\ \mathrm{Im}\,(V_{ud}V_{cd}^* V_{cs} V_{us}^*), \end{aligned} \tag{2}$$

where m_i is the mass of the quark i and V_{ij} is the ij entry of the CKM matrix, implies that CP is conserved if some up or down quark masses are degenerate or a product $V_{ij}V_{kj}^*V_{kl}V_{il}^*$, $i \neq j$, $k \neq l$ is real. On the other hand, any CP violating observable is proportional to the factors in Eq. (2) which then give the size of CP violation (*ii*). This invariant formulation also allows to decide in any weak basis if CP is conserved (*iii*). It also motivates model building (*iv*) and it can eventually help to understand the origin of CP violation if for some reason a definite model (weak basis) is physically distinguished (*v*). In three dimensions [7]

$$\mathrm{tr}\,[M_u M_u^\dagger, M_d M_d^\dagger]^3 = 3I. \tag{3}$$

Hence this trace is also imaginary (see Eq. (2)) and proportional to the area A of a triangle with sides $V_{ud}V_{cd}^*$, $V_{us}V_{cs}^*$, $V_{ub}V_{cb}^*$ and angles ϕ_{1-3} with

$$\begin{aligned}
\sin\phi_1 &= |\sin\arg(V_{ud}V_{cd}^*V_{cs}V_{us}^*)|, \\
\sin\phi_2 &= |\sin\arg(V_{us}V_{cs}^*V_{cb}V_{ub}^*)|, \\
\sin\phi_3 &= |\sin\arg(V_{ub}V_{cb}^*V_{cd}V_{ud}^*)|
\end{aligned}$$

in the complex plane (see Fig. 1) [8, 9],

$$\mathrm{tr}\,[M_u M_u^\dagger, M_d M_d^\dagger]^3 \propto \mathrm{Im}\,(V_{ud}V_{cd}^*V_{cs}V_{us}^*) = 2A. \tag{4}$$

Hence the vanishing of any angle (side) of this triangle stands for CP conservation. A b factory will help to determine the triangle better and to verify if the pattern of CP violation corresponds to the existence of three families only [10]. (In this case other equivalent triangles resulting from the unitarity of the CKM matrix and involving the b quark may be more convenient [4, 5, 10].)

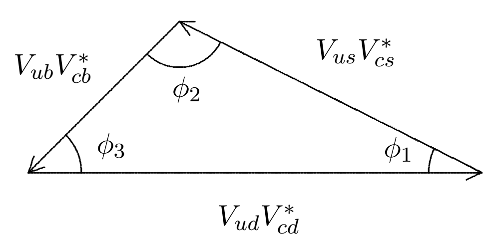

Figure 1: Unitarity triangle

In this paper we revise the analogous formulation for four quark families. There is a large literature on the subject and many of our results have been first obtained by other authors [9, 11]. We have taken advantage of the extensive use of symbolic programs as *Mathematica* [12] for the detailed proofs and for obtaining explicit results [13]. Very often it was necessary educated guessing to avoid otherwise unmanageable intermediate expressions. When this was possible it could be traced back to a deeper, sometimes already known reason. A set of conditions independent of the choice of weak quark basis and necessary and sufficient for CP conservation for four generations was found for nondegenerate quark masses in Ref. [11]. First we extend this set to also include the case of degenerate quark masses. Afterwards we write these invariant quantities as a function of physical parameters, *i.e.* quark masses and products of CKM matrix elements independent of the choice of the phases of quark mass eigenstates. Then the requirement that the area of the triangle in Eq. (4) vanishes if CP is conserved for three families is generalized to the vanishing of the areas of three quadrangles for four generations [9]. Finally we comment on the realistic case of small mixings.

2 Complete set of invariant constraints for CP conservation.

In the standard model with N families the quark mass term can be written, after spontaneous symmetry breaking,

$$-\mathcal{L}_{mass} = \bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \text{h.c.}, \tag{5}$$

where $u_{L,R}$, $d_{L,R}$ are weak eigenstates. This Lagrangian is invariant under a CP transformation leaving the $\mathrm{SU(2)_L \times U(1)_Y}$ gauge interactions unchanged [7]

$$\begin{aligned} u_L &\to U_L C u_L^* \quad , \quad u_R \to U_R^u C u_R^* \ , \\ d_L &\to U_L C d_L^* \quad , \quad d_R \to U_R^d C d_R^* \ , \end{aligned} \tag{6}$$

where C is the Dirac charge-conjugation matrix and U_L, $U_R^{u,d}$ are 4×4 unitary matrices, if

$$U_L^\dagger M_u U_R^u = M_u^* \ , \quad U_L^\dagger M_d U_R^d = M_d^* \ . \tag{7}$$

Thus CP is conserved if U_L, $U_R^{u,d}$ exist fulfilling Eq. (7). In the standard model with only one Higgs doublet $U_R^{u,d}$ are unobservable. Hence, as an arbitrary complex matrix can be written as the product of a hermitian matrix with non-negative eigenvalues times a unitary matrix, $M_{u,d}$ can be assumed to be hermitian with non-negative eigenvalues. Then,

$$U_L^\dagger H_u U_L = H_u^* \ , \quad U_L^\dagger H_d U_L = H_d^*, \tag{8}$$

where $H_u = M_u M_u^\dagger$, $H_d = M_d M_d^\dagger$, are also necessary and sufficient conditions for CP conservation. Under unitary transformations of the left-handed fields the trace of any product of matrices $H_{u,d}$ is invariant. Therefore, Eq. (8) implies

$$\text{Im tr } (H_u^{p_1} H_d^{p_2} H_u^{p_3} \cdots H_d^{p_r}) = 0. \tag{9}$$

with $(p_1, \ldots, p_r)$ an arbitrary sequence of positive integers. Eqs. (9), which are independent of the choice of quark basis, are also necessary and sufficient conditions for CP invariance [11]. However, in practice one is interested in finding a minimal subset of these necessary and sufficient conditions. For $N = 3$ there is one such condition necessary and sufficient for CP conservation,

$$\text{Im tr } (H_u^2 H_d H_u H_d^2) = 0, \tag{10}$$

where $\text{Im tr } (H_u^2 H_d H_u H_d^2) = -\frac{1}{6}\text{Im tr } [H_u, H_d]^3$ in Eq. (3). For $N = 4$ there are six conditions for CP invariance for nondegenerate quark masses, defined by the sequences (2,1,1,2), (2,1,1,3), (2,2,1,3), (1,1,1,2,1,3), (3,1,1,2), (3,1,1,3) in Eq. (9) [11]. For the degenerate case two more constraints must be added. In the *Appendix* we prove that a minimal set of these constraints consists of

$$\begin{aligned}
I_1 &= \text{Im tr } (H_u^2 H_d H_u H_d^2) = 0, \\
I_2 &= \text{Im tr } (H_u^3 H_d H_u H_d^2) = 0, \\
I_3 &= \text{Im tr } (H_u^4 H_d H_u H_d^2 - H_u^3 H_d H_u^2 H_d^2) = 0, \\
I_4 &= \text{Im tr } (H_u^5 H_d H_u H_d^2 - H_u^4 H_d H_u^2 H_d^2 + H_u^3 H_d H_u^2 H_d H_u H_d) = 0, \\
I_5 &= \text{Im tr } (H_u^2 H_d H_u H_d^3) = 0, \\
I_6 &= \text{Im tr } (H_u^3 H_d H_u H_d^3) = 0, \\
I_7 &= \text{Im tr } (H_u^2 H_d H_u H_d^4 - H_u^2 H_d^2 H_u H_d^3) = 0, \\
I_8 &= \text{Im tr } (H_u^2 H_d H_u H_d^5 - H_u^2 H_d^2 H_u H_d^4 + H_u H_d H_u H_d^2 H_u H_d^3) = 0.
\end{aligned} \tag{11}$$

The important terms in $I_{3,4,7,8}$ are the sequences (3, 1, 2, 2), (3,1,2,1,1,1), (2,2,1,3), (1,1,1,2,1,3), respectively. The other terms are added to obtain more compact expressions later. We order I_{1-8} conventionally by increasing number of H_d factors because we assume H_u diagonal and H_d hermitian (as it can be done without loss of generality) and in our proof the expressions for the first conditions are simpler with this choice. At any rate the full set is symmetric under the interchange of H_u and H_d. For the nondegenerate case the sets $I_{1-6} = 0$ and $I_{1,2,5-8} = 0$ are both enough to guarantee CP conservation. The second set is essentially the same as in Ref. [11], whereas the first one results from the interchange of H_u and H_d. (In Ref. [11] H_u was assumed to be hermitian and H_d

diagonal. Then it was natural to look at the conditions with smaller number of H_u factors.) Obviously, the symmetry of both sets is due to the impossibility of distinguishing up and down quarks in Eqs. (5,8). That the complete set is necessary and sufficient for CP conservation even for degenerate quark masses must be proven. We obtain this set of constraints looking for all the invariant expressions with $p = p_1 + p_2 + \cdots + p_r \leq 9$ in Eq. (9) and not identically zero. Invariants related by the cyclic property of the trace and/or the hermiticity of $H_{u,d}$ are counted once. We find one (I_1) with $p = 6$; two ($I_{2,5}$) with $p = 7$; six with $p = 8$ including the two terms of $I_{3,7}$ and I_6; and fourteen with $p = 9$, including the three terms of $I_{4,8}$. Then we solve the vanishing conditions with increasing p till we have no CP violating solution left. The set in Eq. (11) is minimal in the sense that any other subset of constraints with lower p is not sufficient to guarantee CP conservation. There are many other choices of complete sets. For instance, for nondegenerate quark masses I_4 in the complete set I_{1-6} could be replaced by I_7 and

$$
\begin{aligned}
I_9 &= \text{Im tr } (H_u^4 H_d H_u H_d^3 - H_u^3 H_d H_u^2 H_d^3), \\
I_{10} &= \text{Im tr } (H_u^3 H_d H_u H_d^4 - H_u^3 H_d^2 H_u H_d^3), \\
I_{11} &= \text{Im tr } (H_u^4 H_d H_u H_d^4 - H_u^4 H_d^2 H_u H_d^3 - H_u^3 H_d H_u^2 H_d^4 + H_u^3 H_d^2 H_u^2 H_d^3)
\end{aligned}
\tag{12}
$$

(see below). The proofs of the completeness of these sets were based in the properties of hermitian matrices in Ref. [11] and in the power of symbolic programs to solve explicitly the constraints here. These conditions satisfy points (*iii*), (*iv*), (*v*) for four families. In order to discuss points (*i*), (*ii*) we have to write these constraints in the mass eigenstate basis, *i.e.* as a function of quark masses and charged current mixings.

3 Invariant formulation of CP violation in the mass eigenstate basis.

If we write $H_u = diag\ (m_1^2, m_2^2, m_3^2, m_4^2)$, $H_d = V_{CKM}\ diag\ (n_1^2, n_2^2, n_3^2, n_4^2)\ V_{CKM}^\dagger$ where 1, 2, 3, 4 stand for u, c, t, t' in H_u and d, s, b, b' in H_d, and V_{CKM} is the unitary matrix diagonalizing H_d, Eqs. (11, 12) read

$$
\begin{aligned}
I_1 &= \sum_{i<j,k<l} G(i,j;k,l), \\
I_2 &= \sum_{i<j,k<l} (m_i^2 + m_j^2 + m_4^2)\ G(i,j;k,l), \\
I_3 &= \sum_{i<j,k<l} (m_i^4 + m_j^4 + m_4^4)\ G(i,j;k,l), \\
I_5 &= \sum_{i<j,k<l} (n_k^2 + n_l^2 + n_4^2)\ G(i,j;k,l),
\end{aligned}
$$

$$
\begin{aligned}
I_6 &= \sum_{i<j,k<l} (m_i^2+m_j^2+m_4^2)\,(n_k^2+n_l^2+n_4^2)\,G(i,j;k,l), \\
I_9 &= \sum_{i<j,k<l} (m_i^4+m_j^4+m_4^4)\,(n_k^2+n_l^2+n_4^2)\,G(i,j;k,l), \\
I_7 &= \sum_{i<j,k<l} (n_k^4+n_l^4+n_4^4)\,G(i,j;k,l), \\
I_{10} &= \sum_{i<j,k<l} (m_i^2+m_j^2+m_4^2)\,(n_k^4+n_l^4+n_4^4)\,G(i,j;k,l), \\
I_{11} &= \sum_{i<j,k<l} (m_i^4+m_j^4+m_4^4)\,(n_k^4+n_l^4+n_4^4)\,G(i,j;k,l),
\end{aligned} \tag{13}
$$

with

$$
\begin{aligned}
G(i,j;k,l) &= -(m_j^2-m_i^2)(m_4^2-m_i^2)(m_4^2-m_j^2)(n_l^2-n_k^2)(n_4^2-n_k^2)(n_4^2-n_l^2) \\
&\times\ \mathrm{Im}\,(V_{ik}V_{jk}^*V_{jl}V_{il}^*).
\end{aligned} \tag{14}
$$

$I_{4,8}$ involve in principle the imaginary parts of products of six CKM matrix elements. However, all of these (96) can be written in terms of only one of them plus products of Im $(V_{ik}V_{jk}^*V_{jl}V_{il}^*)$ times the modulus of a CKM matrix element squared. Although the term with the imaginary part of six V_{CKM} matrix elements cancels in $I_{4,8}$, their expressions are still too long to be written here. We could also try to write Im $(V_{ik}V_{jk}^*V_{jl}V_{il}^*)$ and then I_α as a function of the 3 CP violating phases parametrizing the CKM matrix in the four family case. With this purpose we could use for instance the parametrization of V_{CKM} in Ref. [14]. However, due to the relatively complicated dependence of V_{ij} on these 3 phases the expressions for I_α are too long to write them here. Im $(V_{ik}V_{jk}^*V_{jl}V_{il}^*) = 0$; $i<j$, $k<l$; $i,j,k,l = 1,2,3$ form also a set of necessary and sufficient constraints for CP conservation for four quark generations [15]. These conditions are invariant only under quark eigenstate phase redefinitions. The cancellation of these 9 quantities guarantees that V_{CKM} can be made real and viceversa. Unitarity allows for other choices of 9 conditions, for instance involving also the fourth family. We conventionally choose not to do so at present. Thus Eqs. (13) prove that $I_{1-3,5-7,9-11}$ are necessary and sufficient constraints for CP conservation in the case of nondegenerate fermion masses, because they are linear in Im $(V_{ik}V_{jk}^*V_{jl}V_{il}^*)$ and independent. Now points (i), (ii) can be answered: CP is conserved if there are three up (down) degenerate masses, three pairs of degenerate masses or the 9 quantities Im $(V_{ik}V_{jk}^*V_{jl}V_{il}^*)$ are zero. (In the degenerate case one also requires $I_{4,8}=0$.)

4 CP violating effects of a fourth family.

Im $V_{ik}V^*_{jk}V_{jl}V^*_{il}$ can be measured in the mass eigenstate basis and one of them at least must be non-zero if CP is violated. This can be summarized with three quadrangles in the complex plane. The sides of the quadrangles are for example the products of the elements of the first line of the CKM matrix times the elements of the second line complex conjugate, whose sum is zero by unitarity (see Fig. 2)

$$V_{ud}V^*_{cd} + V_{us}V^*_{cs} + V_{ub}V^*_{cb} + V_{ub'}V^*_{cb'} = 0. \tag{15}$$

The area of the convex quadrangle drawn with these products is

$$\begin{aligned} A_{uc} &= \frac{1}{4}\{|\mathrm{Im}\ (V_{ud}V^*_{cd}V_{cs}V^*_{us})| + |\mathrm{Im}\ (V_{us}V^*_{cs}V_{cb}V^*_{ub})| + |\mathrm{Im}\ (V_{ub}V^*_{cb}V_{cb'}V^*_{ub'})| \\ &+ |\mathrm{Im}\ (V_{ub'}V^*_{cb'}V_{cd}V^*_{ud})|\}, \end{aligned} \tag{16}$$

whereas the angles are the arguments of the four-products,

$$\begin{aligned} \sin\phi_1 &= |\sin\arg(V_{ud}V^*_{cd}V_{cs}V^*_{us})|, \\ \sin\phi_2 &= |\sin\arg(V_{us}V^*_{cs}V_{cb}V^*_{ub})|, \\ \sin\phi_3 &= |\sin\arg(V_{ub}V^*_{cb}V_{cb'}V^*_{ub'})|, \\ \sin\phi_4 &= |\sin\arg(V_{ub'}V^*_{cb'}V_{cd}V^*_{ud})|. \end{aligned}$$

Hence the vanishing of the area of the quadrangle means that the four terms in Eq. (16) vanish and viceversa. One can consider also the quadrangles and areas resulting from multiplying the first and third rows or the second and third ones. Thus the vanishing of $A_{uc,ut,ct}$ means that the corresponding 12 terms on the right-hand side vanish, and it is easy to convince oneself using the unitarity of the CKM matrix that this is equivalent to the cancellation of the 9 independent quantities Im $(V_{ik}V^*_{jk}V_{jl}V^*_{il})$ involving only the first three families and therefore to CP invariance [15]. A numerical update of the present limits for a fourth family will be presented elsewhere. For illustration purposes, however, let us assume that the thesis in Ref [16] holds and

$$|V_{CKM}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 & \lambda^2 \\ \lambda & 1 & \lambda^2 & \lambda \\ \lambda^3 & \lambda^2 & 1 & \lambda \\ \lambda^2 & \lambda & \lambda & 1 \end{pmatrix} \tag{17}$$

with $\lambda = |V_{us}| = 0.22$ [17]. In this case the quadrangles have areas $A_{uc} \sim \lambda^4$, $A_{ut} \sim \lambda^6$, $A_{ct} \sim \lambda^4$, respectively. If the fourth family does not mix, $V_{ib'} = V_{t'j} = 0$, $i = u, c, t$; $j = d, s, b$, the three quadrangles collapse to three triangles (shaded region of the quadrangle in Fig. 2) with the same area $A \sim \lambda^6$ in Eq. (4).

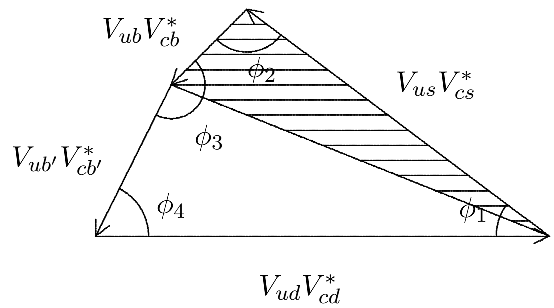

Figure 2: Unitarity quadrangle

A Appendix.

H_u is diagonal, $(H_u)_{ij} = m_i^2 \delta_{ij}$, with m_i the mass of the up quark i, and H_d hermitian with $(H_d)_{ij} = n_{ij}$, $n_{ij} = n_{ji}^*$. Let us define $d_{ij} \equiv m_j^2 - m_i^2$, $C(i,j,k) \equiv d_{ij} d_{jk} d_{ki}$ Im $n_{ij} n_{jk} n_{ki}$, then

$$
\begin{aligned}
I_1 &= \sum_{i<j<k} C(i,j,k), \qquad (18)\\
I_2 &= \sum_{i<j<k} (m_i^2 + m_j^2 + m_k^2)\, C(i,j,k),\\
I_3 &= \sum_{i<j<k} (m_i^4 + m_j^4 + m_k^4)\, C(i,j,k),\\
I_4 &= \sum_{i<j<k} (m_i^6 + m_j^6 + m_k^6)\, C(i,j,k).
\end{aligned}
$$

The determinant of the coefficients of the C's is just $\prod_{i<j}(m_j^2 - m_i^2)$, so in the case of nondegenerate quark masses $I_{1-4} = 0$ imply $C(i,j,k) = 0$ and all the three-cycles $n_{ij} n_{jk} n_{ki}$ are real. However, there are still CP violating solutions with $n_{ij} = n_{kl} = 0$, where i, j, k, l are four distinct indices. In Ref. [11], this is identified as the only case in which the reality of the three-cycles does not imply the reality of the four-cycles $n_{ij} n_{jk} n_{kl} n_{li}$. Due to the symmetry of the problem, we can assume $n_{12} = n_{34} = 0$. In this case all the four-cycles but $n_{13} n_{32} n_{24} n_{41}$ are real. If $n_{12} = n_{34} = 0$,

$$
\begin{aligned}
I_5 &= d_{12}\, d_{34}\, (m_1 + m_2 - m_3 - m_4)\ \mathrm{Im}\ n_{13} n_{32} n_{24} n_{41},\\
I_6 &= d_{12}\, d_{34}\, (m_1^2 + m_1 m_2 + m_2^2 - m_3^2 - m_3 m_4 - m_4^2)\ \mathrm{Im}\ n_{13} n_{32} n_{24} n_{41},
\end{aligned}
$$

then $I_{5,6} = 0$ imply Im $n_{13}n_{32}n_{24}n_{41} = 0$ and CP conservation.

In the case of degenerate masses we can assume without loss of generality $m_1^2 = m_2^2$, $n_{12} = 0$. In this case I_{2-4} are proportional to I_1,

$$I_5 = (n_{11} + n_{33} + n_{44})\, C(1,3,4) + (n_{22} + n_{33} + n_{44})\, C(2,3,4)$$

and I_6 is a linear combination of $I_{1,5}$. We can distinguish two subcases. If $n_{11} = n_{22}$, we can transform H_d and assume $n_{23} = 0$. Then $I_1 = 0$ guarantees CP conservation. If $n_{11} \neq n_{22}$, I_1 and I_5 still have two CP violating solutions corresponding to $n_{34} = 0$ and $m_3^2 = m_4^2$. In the latter case we can transform H_d and also assume $n_{34} = 0$. If $m_1 = m_2$, $n_{12} = n_{34} = 0$,

$$\begin{aligned} I_7 &= d_{13}\, d_{34}\, d_{41}\, (n_{11} - n_{22}) \,\mathrm{Im}\, n_{13}n_{32}n_{24}n_{41}, \\ I_8 &= d_{13}\, d_{41}\, (n_{11} - n_{22})(d_{14}n_{44} - d_{13}n_{33}) \,\mathrm{Im}\, n_{13}n_{32}n_{24}n_{41}, \end{aligned}$$

and $I_{7,8} = 0$ ensure CP conservation.

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